# 1 Background Theory and Experimental Methods

## 1.1 DC Motor Systems Modeling



Figure : DC Brush Motor Electro-mechanical model

The electro-mechanical model of a DC brush motor is shown in **figure 1**. The electrical circuit takes an input voltage and generates a voltage drop across the rotor. The electrical energy from the voltage drop can be related to the resulting angular velocity, , through the motor velocity constant, :

(Equation 1)

The current of the circuit, , can also be coupled to the resulting torque in the motor,, through the back-EMF constant, :

(Equation 2)

Performing electrical and mechanical analysis on the system using equations 1 and 2 leads to the following transfer function relating the output angular velocity to the input voltage:

(Equation 3)

Equation 3 is typically written in the following format for ease of identifying the system parameters, system gain, , and time constant, :

Therefore, the equations for the system gain and time constant of a motor are:

(Equation 5)

(Equation 4)

The detailed derivation steps for the motor transfer function are attached in Appendix A1.1 in this document.

## 1.2 Experimental Setup and Methods

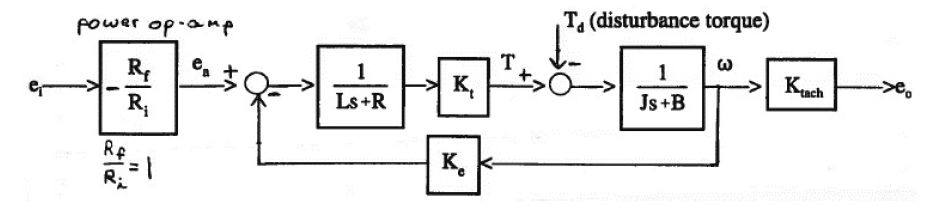


Figure : Experimental Setup Block Diagram

The full experimental setup is shown in the block diagram in **figure 2**. The generated input voltage is passed through a power op-amp with a gain of to produce the motor input voltage . The motor’s resulting angular velocity is fed back to complete a closed-loop feedback system through the motor velocity constant. The difference (error) between the input voltage and the output angular velocity controls the motor. The term represents the electrical system’s resulting voltage drop at the rotor, which converts the electrical energy into torque through the back-EMF constant. Potential disturbance loading is included in the schematic. The angular velocity is measured by a tachometer with a gain of and outputs a voltage measurable with a scope.

It is worth noting that when the disturbance load is equal to the generated torque from a given voltage, , the input into the mechanical model would be zero, resulting in zero angular velocity. But the motor is still generating a torque . This torque is the stall torque at the given voltage .

A back-drive motor is connected to the motor under test (MUT). In the first part, the back-drive motor delivers torque to the MUT, converting the motor into a generator, and the generated voltage and the tachometer output for the back-drive motor are measured. The data is used to determine the back-EMF constant, , of the MUT by linearly fitting the angular velocity with the MUT output, assuming the loss between the back-drive and MUT connection is negligible.

In the second part, after is determined, the MUT is directly connected to determine the motor’s time constant, gain, and response to disturbance load. The steady-state error of the disturbance response is the difference between the output before and after engaging the disturbance.

Finally, the system is then connected to proportional, integral, and proportional-integral controllers to form a closed-loop system. The three controllers’ circuit diagram is shown in **figure 3** below. The P, I, and PI controllers’ transfer functions are shown in equations 6 to 8. The P controller’s gain is 1.5:

(Equation 6)

(Equation 8)

(Equation 7)

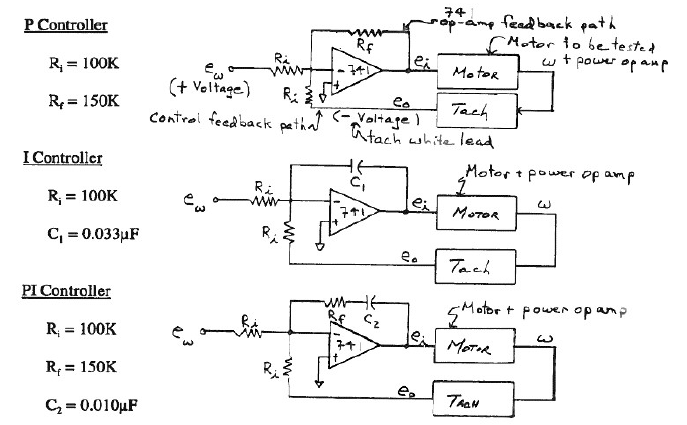


Figure : P, I, and PI Controller circuit diagrams showing connection to the rest of the system

# Appendix

## A1.1 DC Motor Electromechanical Model Derivation

Given the following system for a DC motor:

From the circuit, perform Kirchhoff’s loop law:

The voltage drop is proportional to the resulting angular velocity of the mechanical system, and the current is proportional to the resulting torque by:

Using and , the mechanical system equation is:

Since the inductance is three magnitudes less than the resistance, the inductance term is neglected. Laplace transform and combine the equations:

The transfer function relating output angular velocity to input voltage is then:

## A1.2 Controller transfer function derivations

Given the P controller setup shown in **figure 3**, assuming ideal op-amp, performing nodal analysis on the negative lead node gives:

Ideal op-amp means , and is grounded to zero. Therefore:

Rearranging:

Similarly, for I controller:

The PI controller is slightly more complicated. The resistor-capacitor in series need to be combined into one element:

The repeat the same steps: